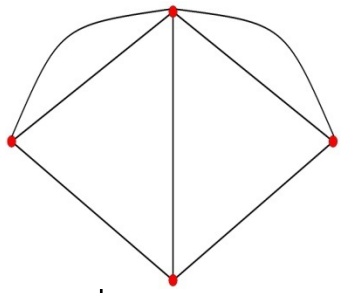
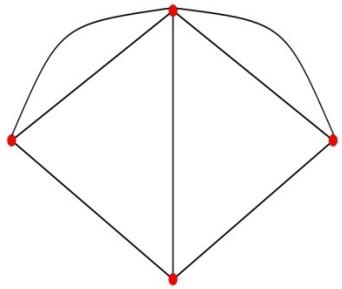


Network Structure and the Speed of Communication

USC April 14, 2009

Matthew O. Jackson
Collaborators: Benjamin Golub, Sergio Currarini, Paolo Pin

Introduction

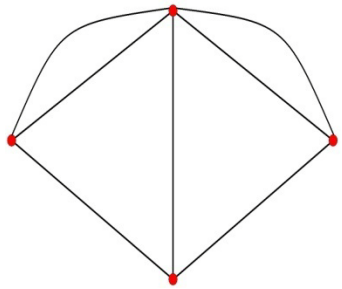


Social Economics: decision making/interactions in embedded settings

Network Structure → Behavior → Network Structure

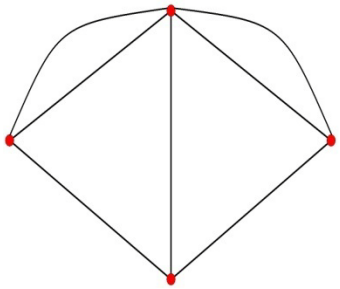
Network Structure → Information Flow/Consensus

An Agenda



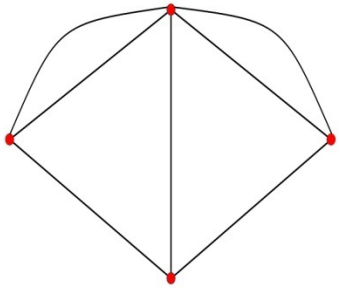
- See which aspects of network structure matter?
- How does the impact of network structure depend on the behavioral process?

A Basic Characteristic of Social (Human) Systems:



- **Homophily**
  - Bias of relationships towards own type
- How does homophily impact
  - diffusion?
  - learning?
  - Behaviors…
- Technology is changing number and patterns of human interactions…

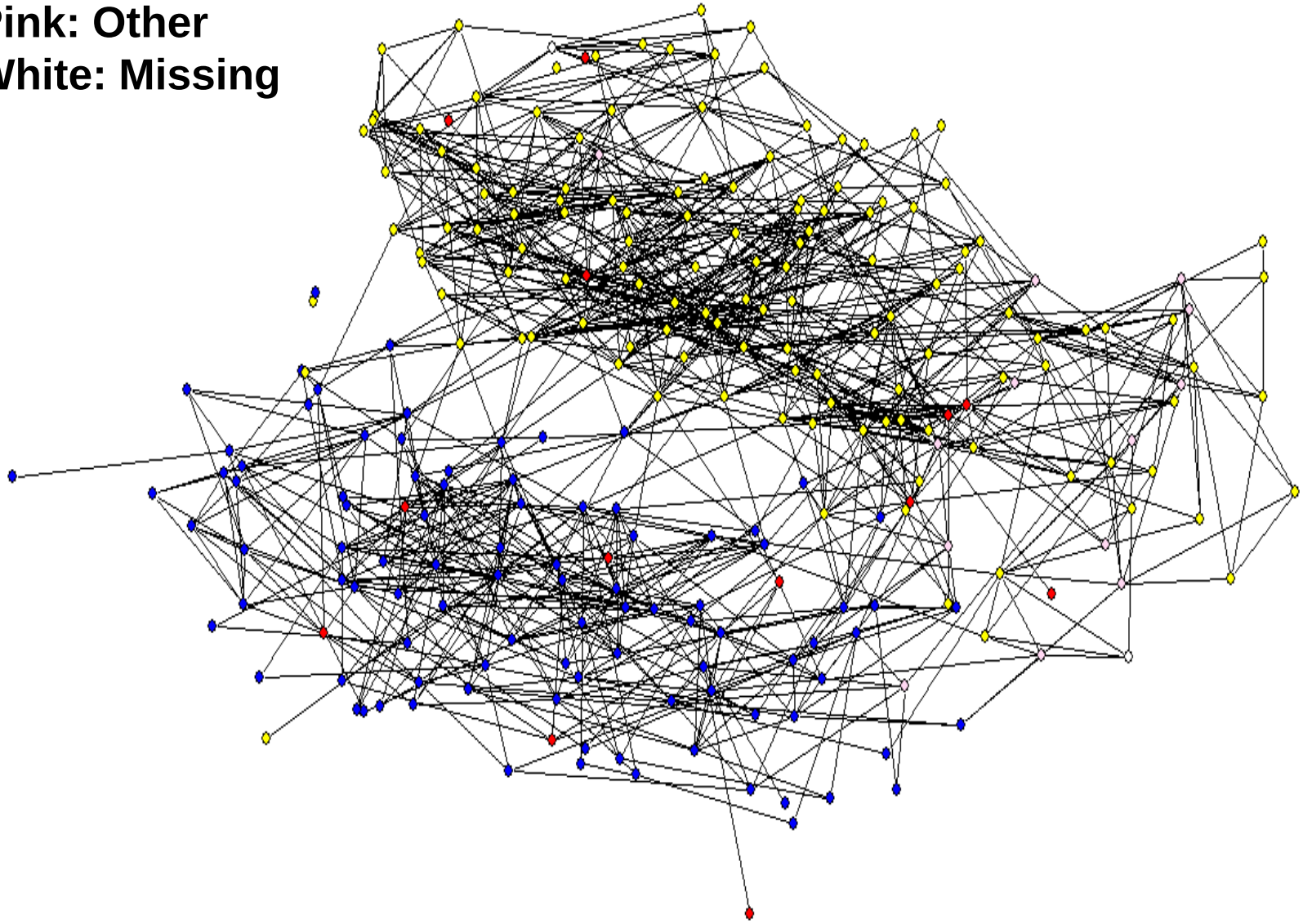
Homophily:



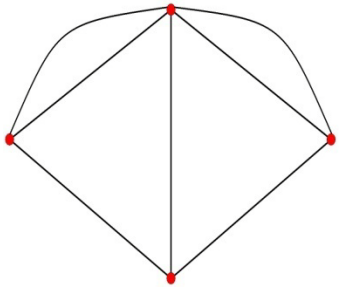
- Tendency to associate with others with similar characteristics: age, race, gender, religion, profession….
  - Lazarsfeld and Merton (1954) ``Homophily''
  - Shrum (gender, ethnic, 1988…), Blau (professional 1974, 1977), Burt, Marsden (variety, 1987, 1988), Moody (grade, racial, 2001…), McPherson (variety,1991…)…

Yellow: Whites
Blue: Blacks
Reds: Hispanics
Green: Asian
Pink: Other
White: Missing

Currarini, Jackson, Pin 2008



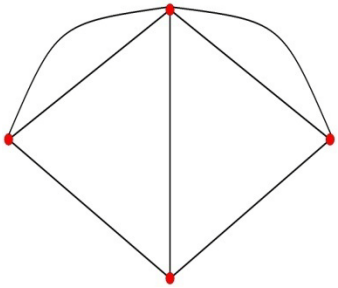
Uncertainty



- How does the reaching of consensus / resolution of uncertainty depend on network structure

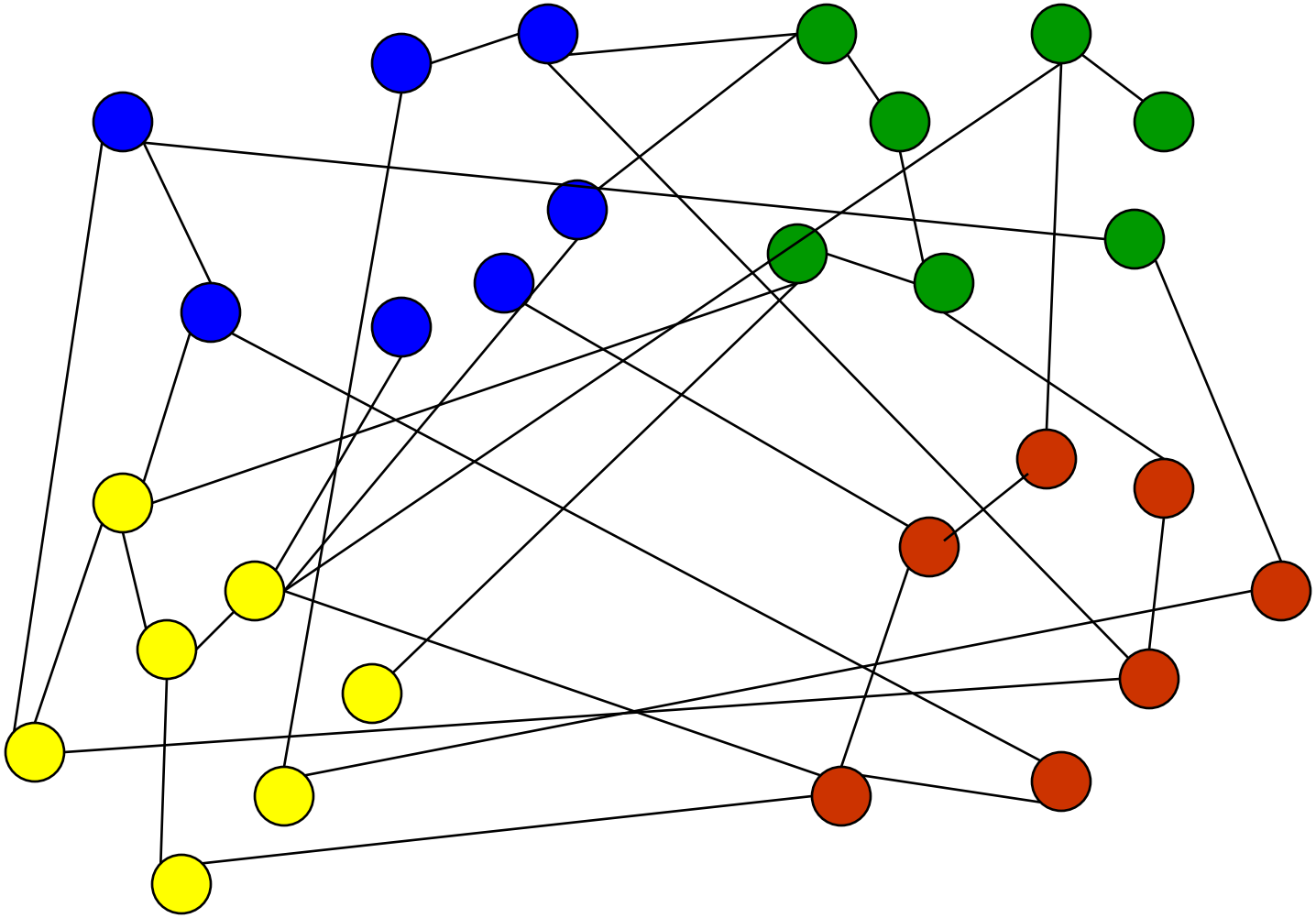
- How does the reaching of consensus / resolution of uncertainty depend on the process

Simple Random Network Model with Homophily

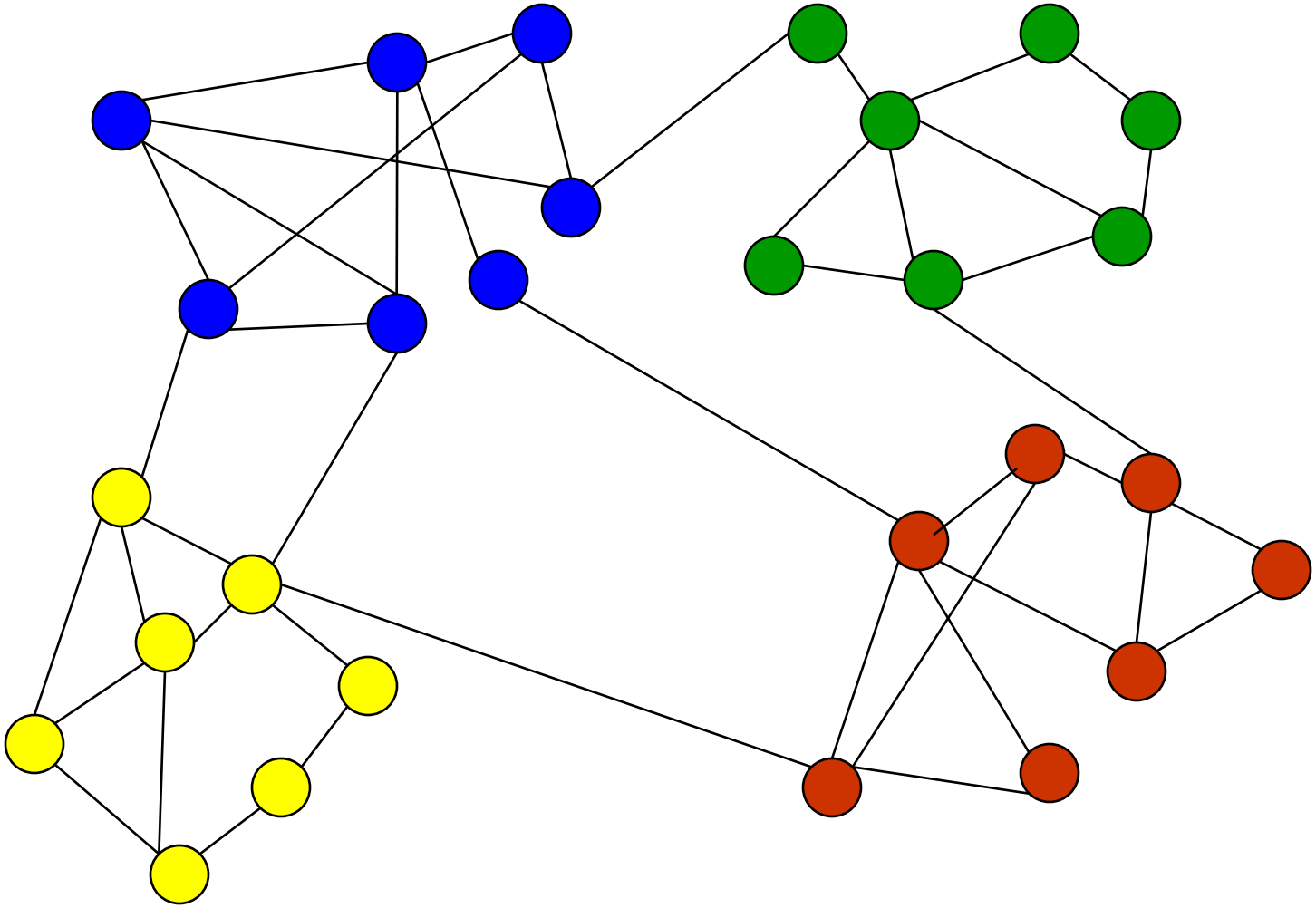


- {1, …, n} agents/nodes
- Partitioned into groups N_1, …, N_m
- Node i in group k is linked to a node j in group k' with probability $P_{kk'}$ (undirected)
- Homophily: $P_{kk} > P_{kk'}$ for $k' \neq k$

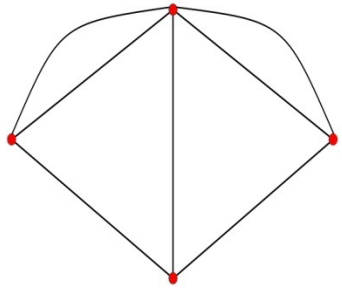
Example Low Homophily



Example High Homophily



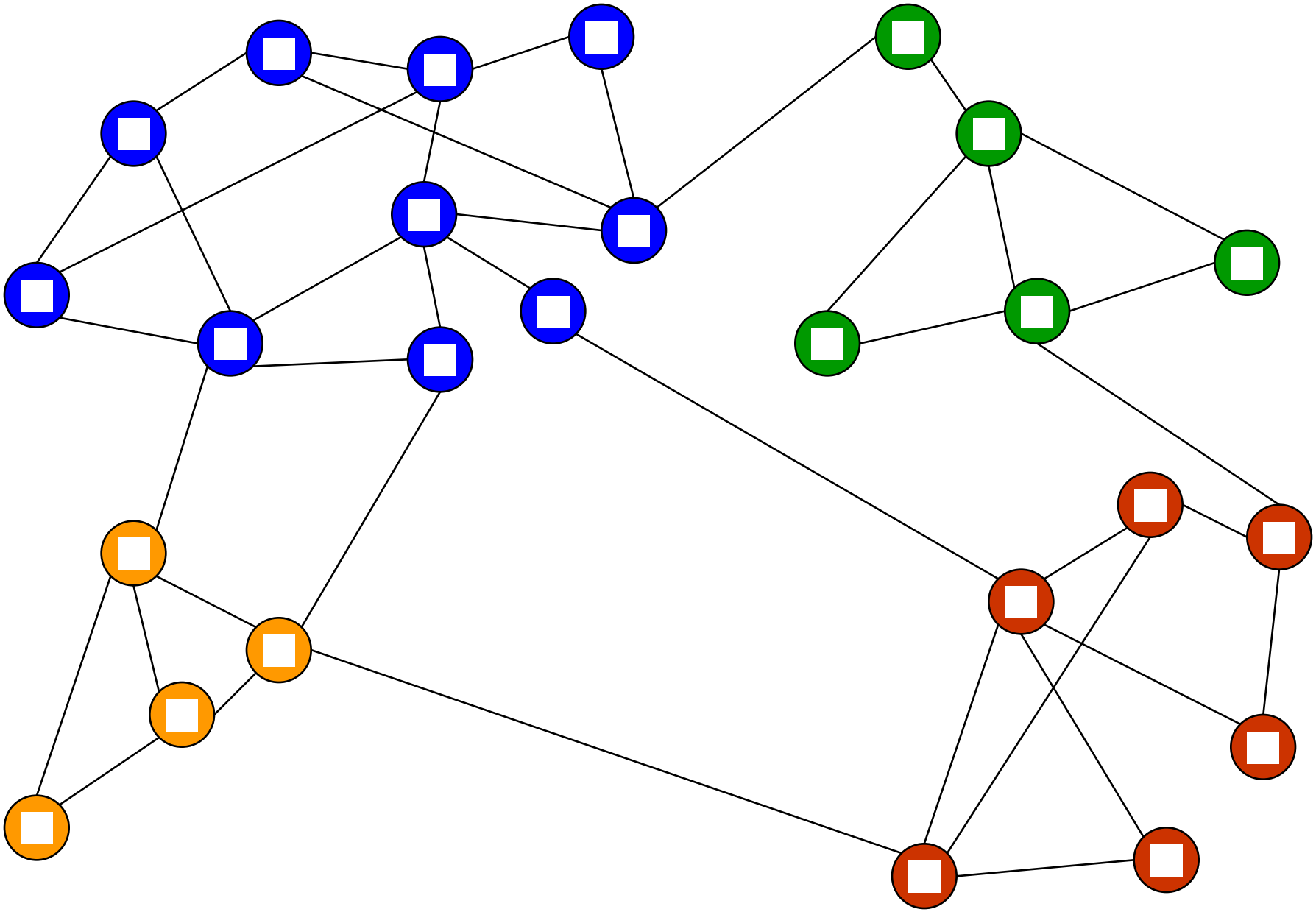
Diffusion/Learning Processes:

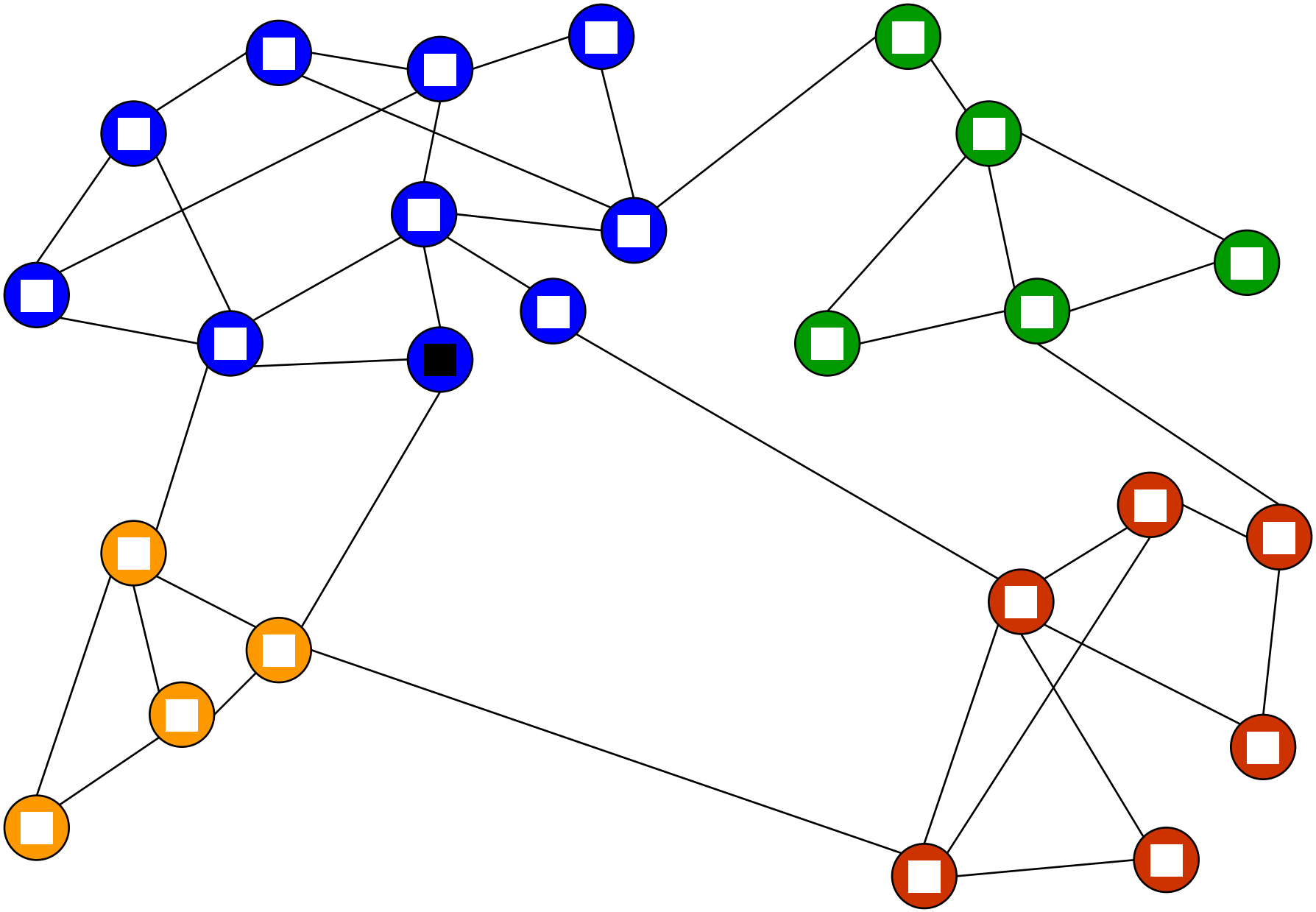


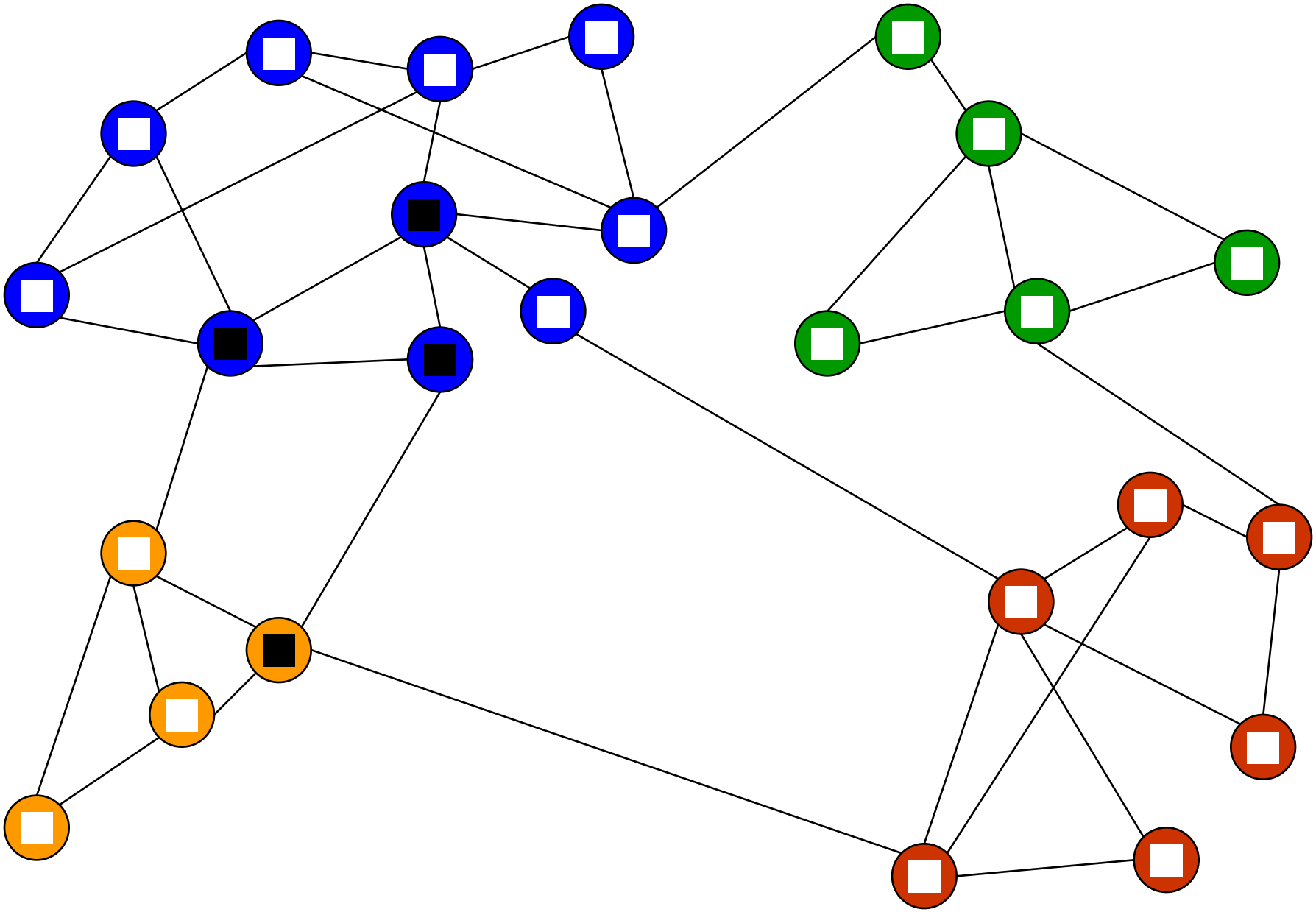
(1) Diffusion / gossip / broadcasting: speed depends on length of shortest paths

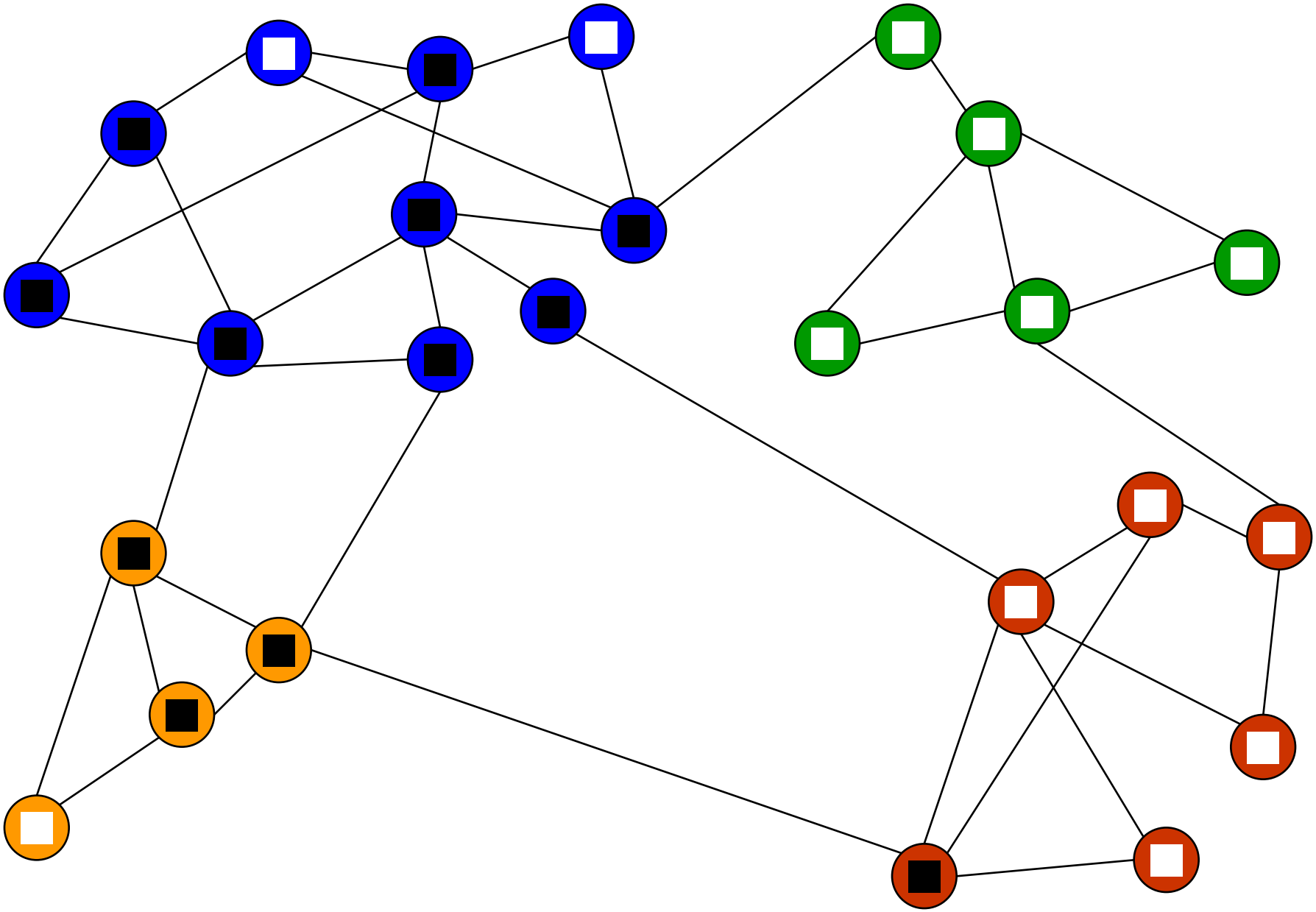
(2) Learning: repeated discussion and weighing of neighbors' opinions

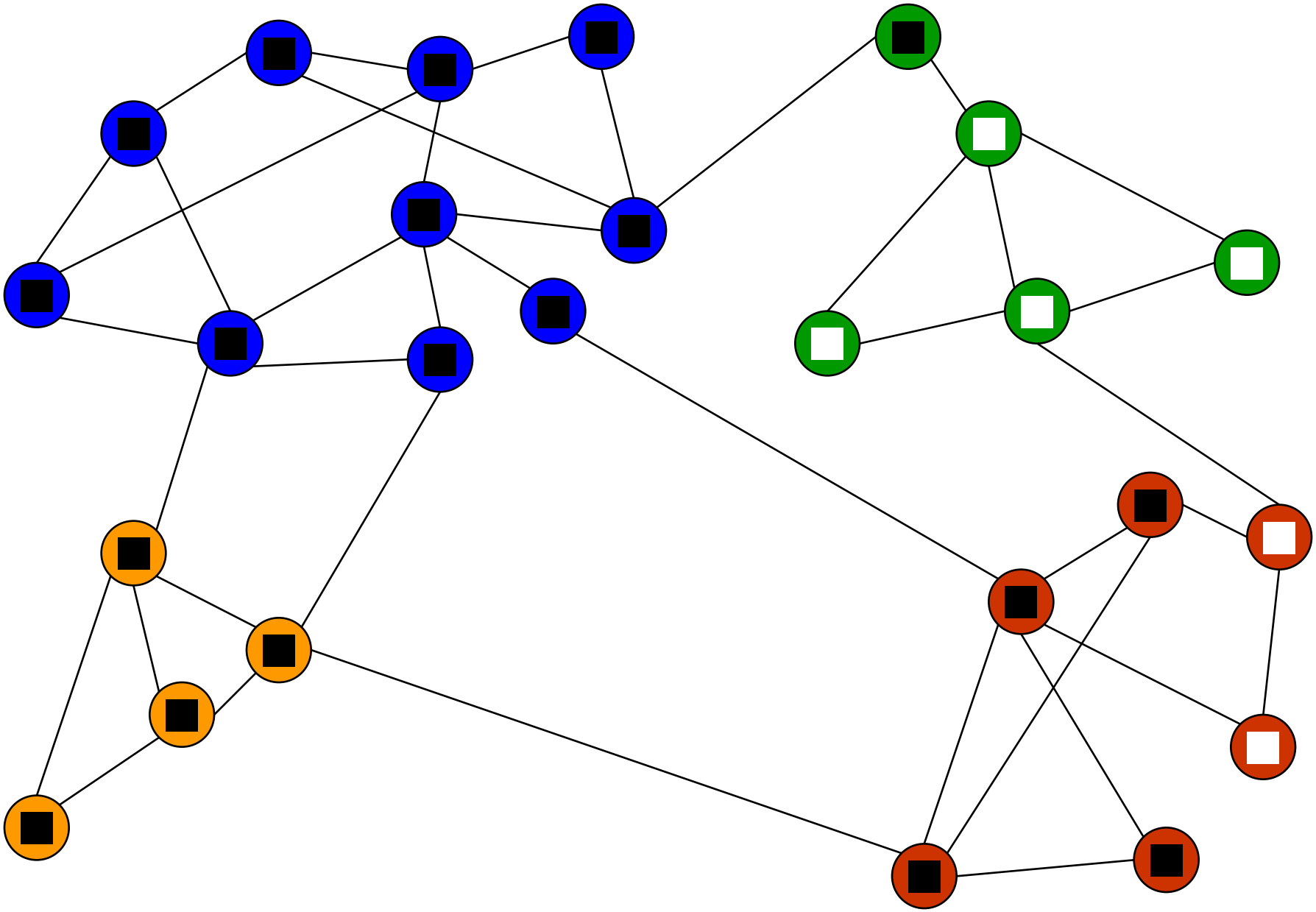
1. Diffusion Speed/ Shortest Paths:

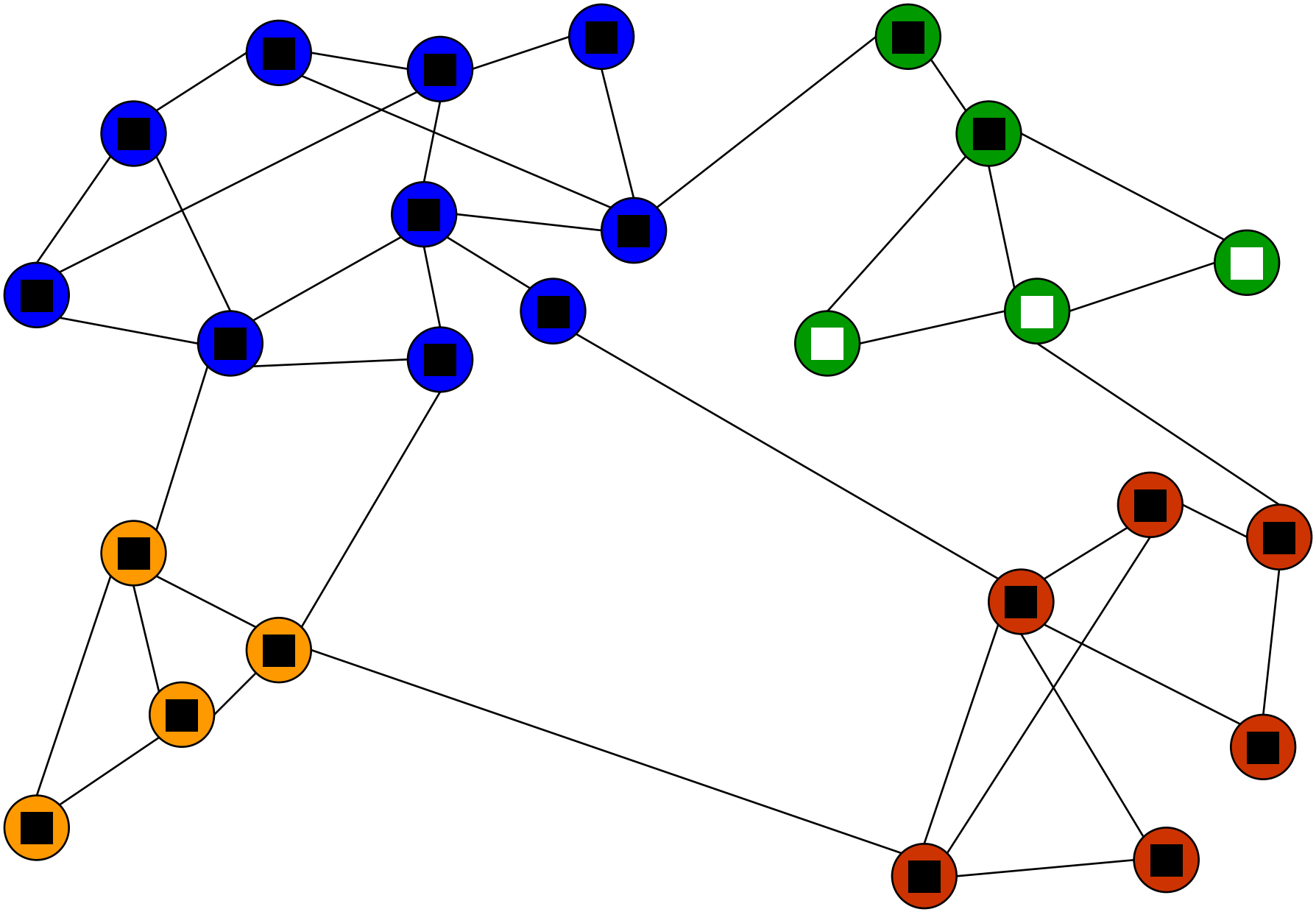


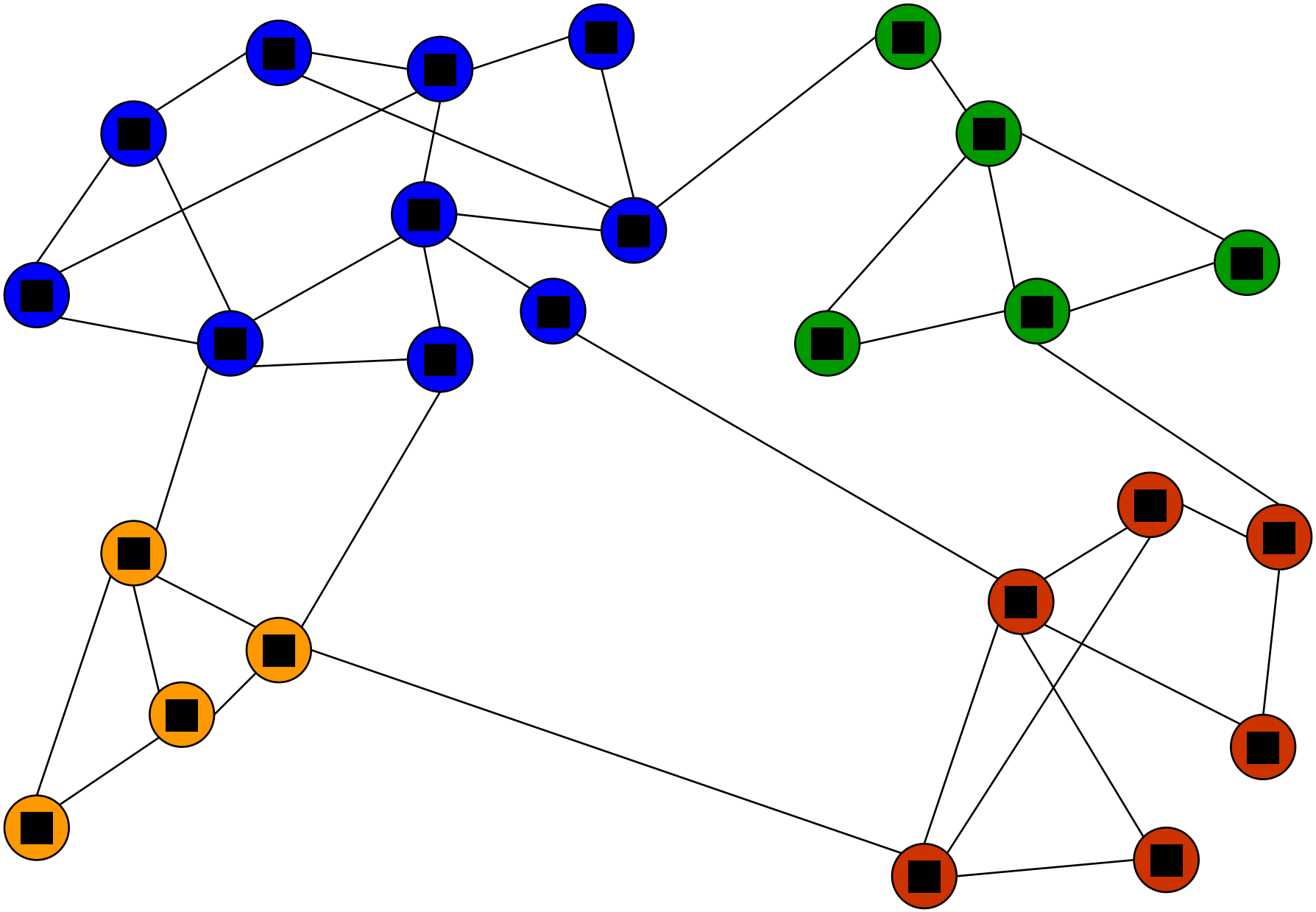








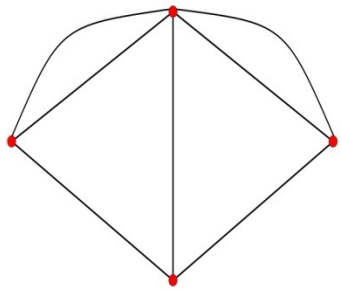




2. Updating/Learning Model
French (1956), Harary (1959), DeGroot (1974), Friedkin and Johnsen (1990)

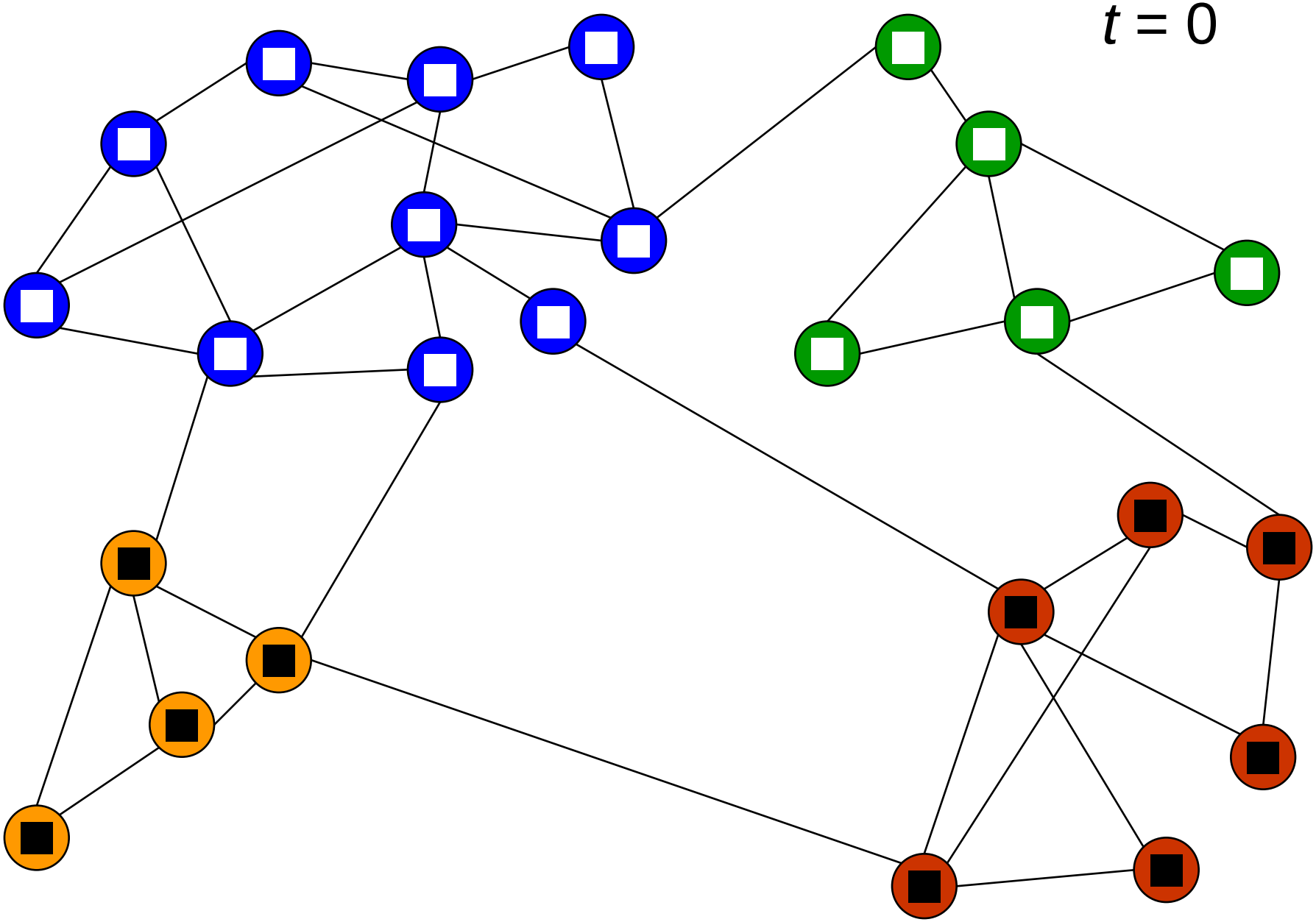
- At each date talk to neighbors
- update opinion or behavior by taking an average of neighbors' opinion/behavior
- Iterate on this process

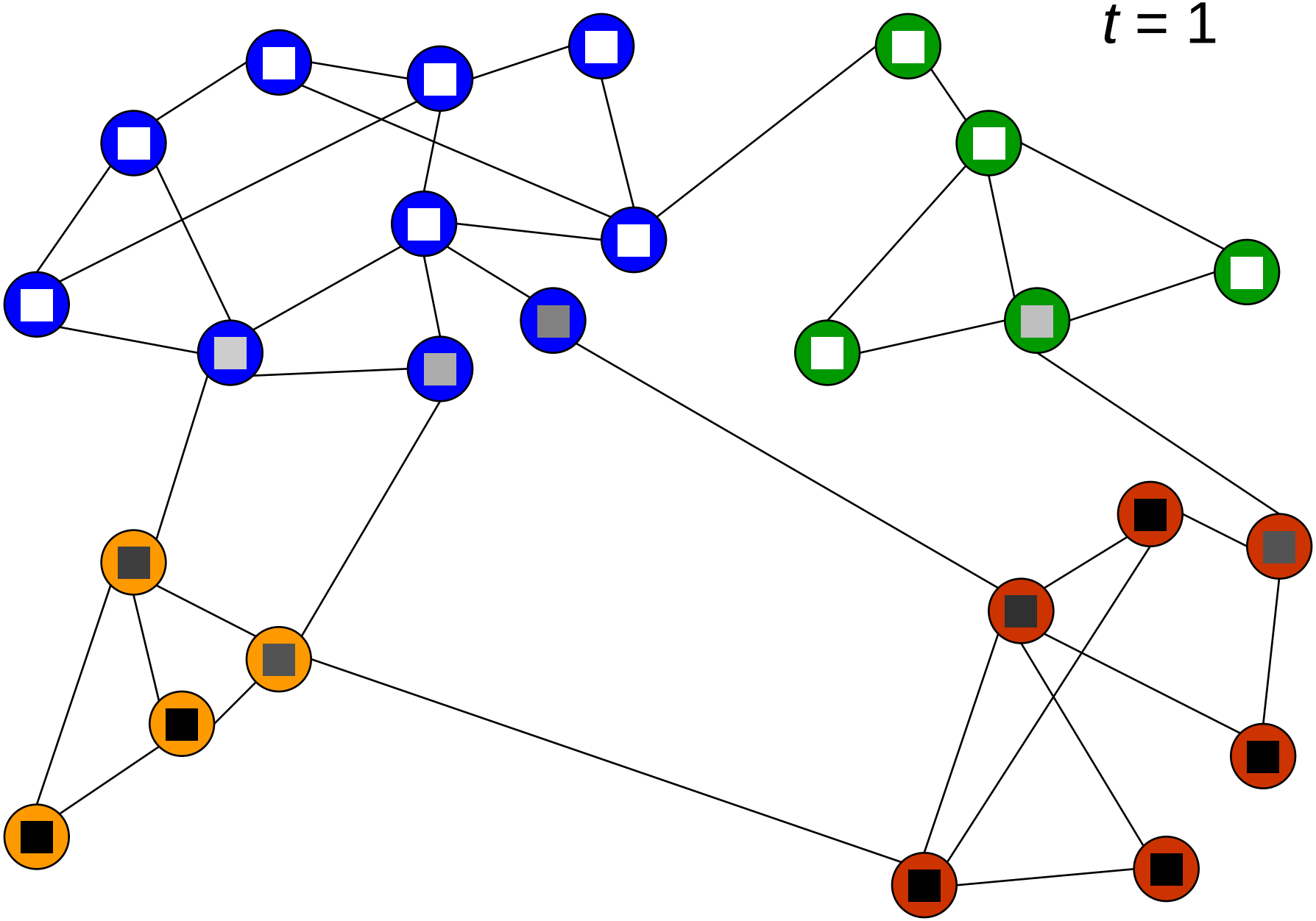
Updating Model

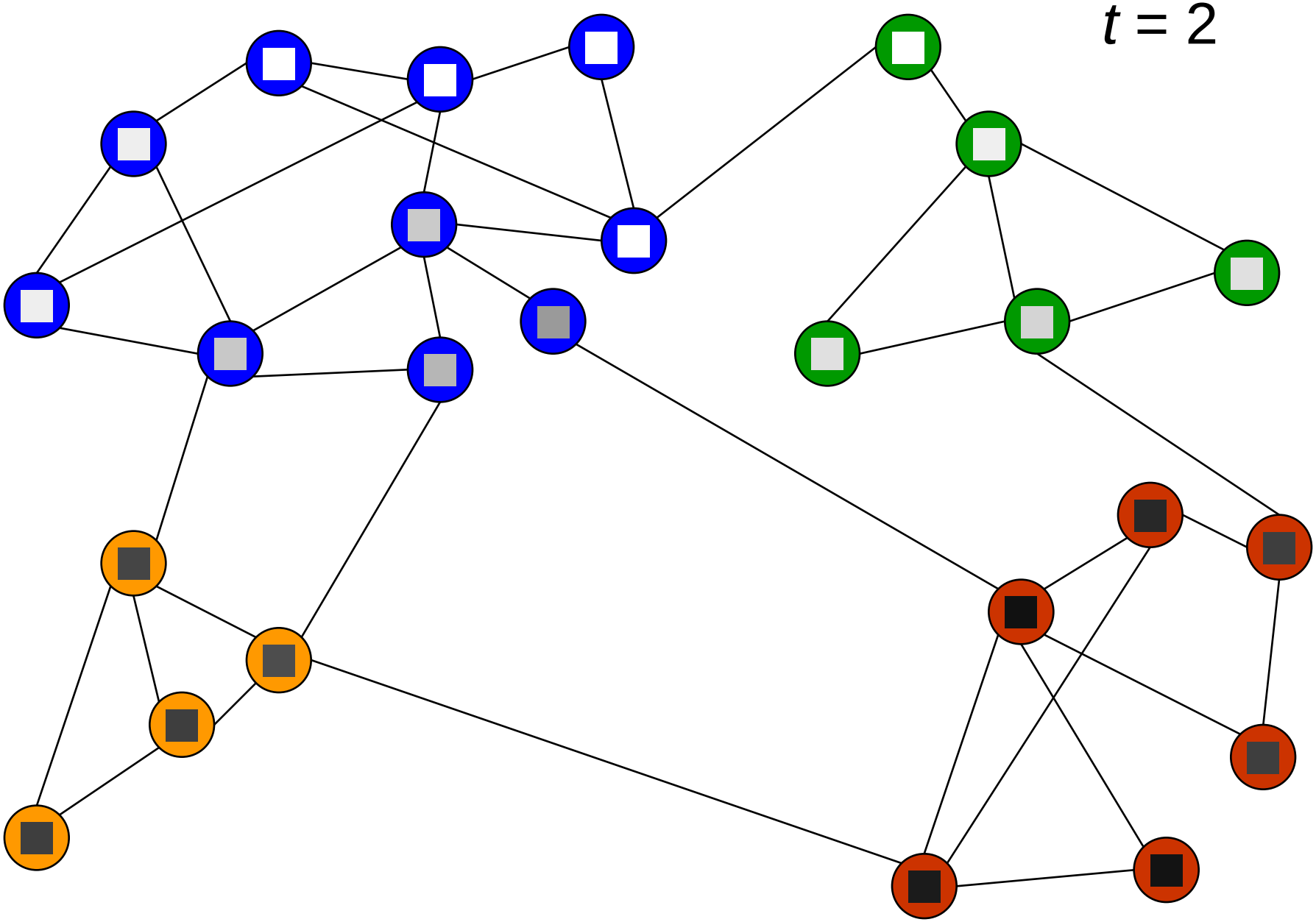


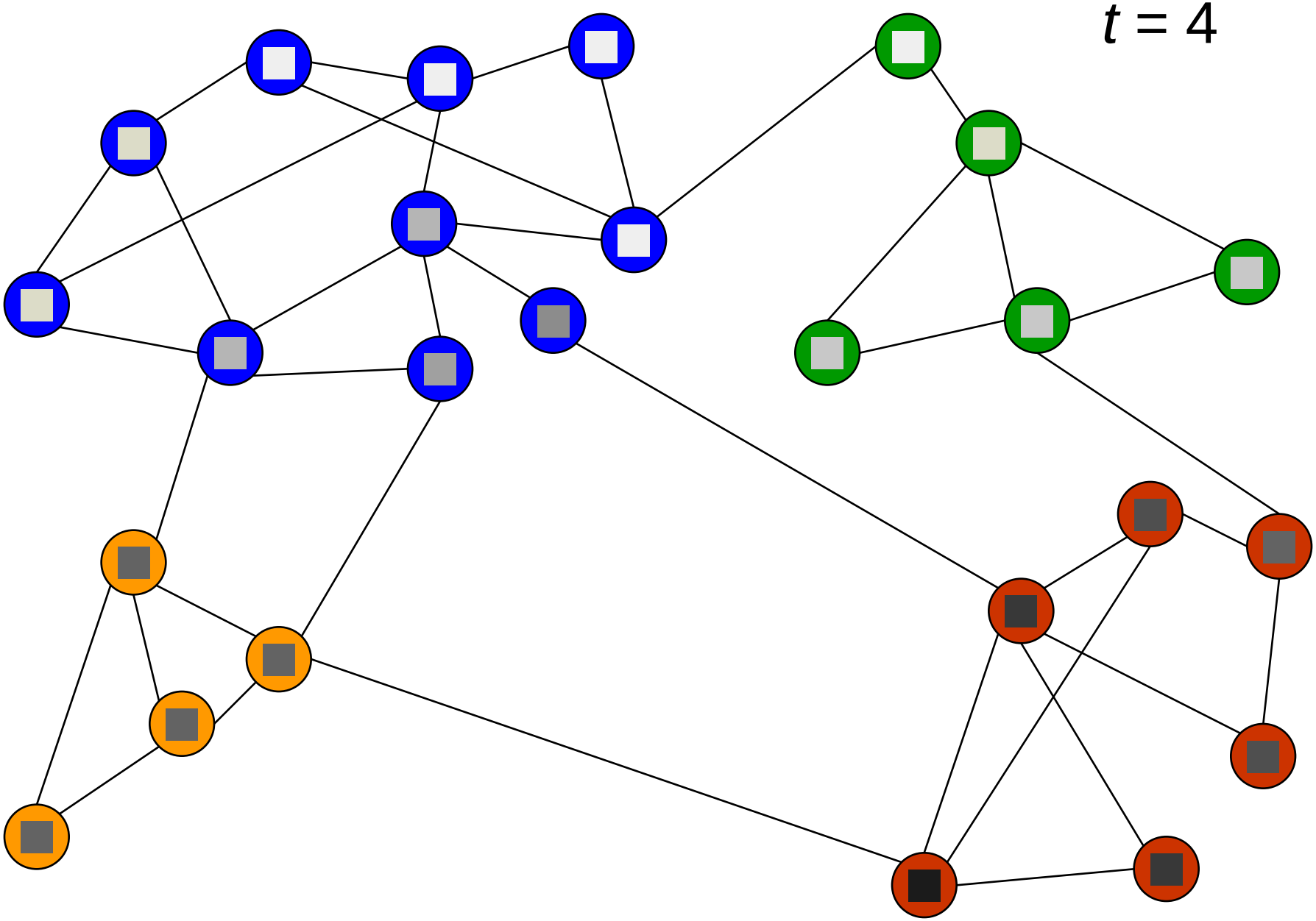
- Individuals {1, ... n} are in a network **A**
- $T_{ij}(\mathbf{A}) = A_{ij}/d_i(\mathbf{A})$ weight that i puts on j
- Start with beliefs (behavior, etc.) $b_i(0)$ in [0,1]
- Updating: $b_i(t) = \sum_j T_{ij}\, b_j(t\text{-}1)$

  So: $b(t) = \mathbf{T}\, b(t\text{-}1) = \mathbf{T}^t\, b(0)$

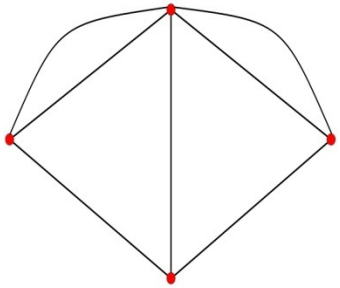

$t = 0$


t = 1


t = 2


t = 4

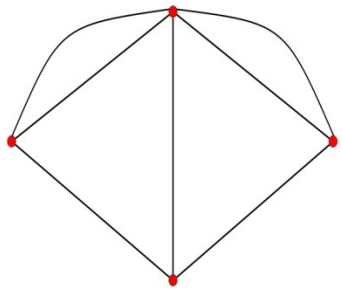
Consensus Time



$$CT(T,\varepsilon) = \sup_b \min \{t: ||T^t b - T^\infty b|| < \varepsilon\}$$

How long until vector of beliefs is within ε of its limit? (worst case)

Diffusion Analysis (Jackson 08)

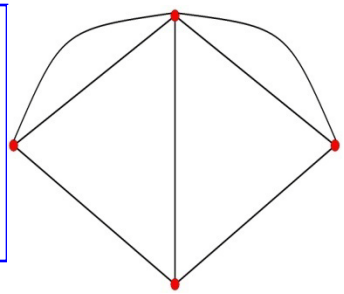


If network is connected:

$$\frac{\text{AvgDist}}{\log(n)/\log(d)} \rightarrow^{P} 1$$

link density matters but not homophily!

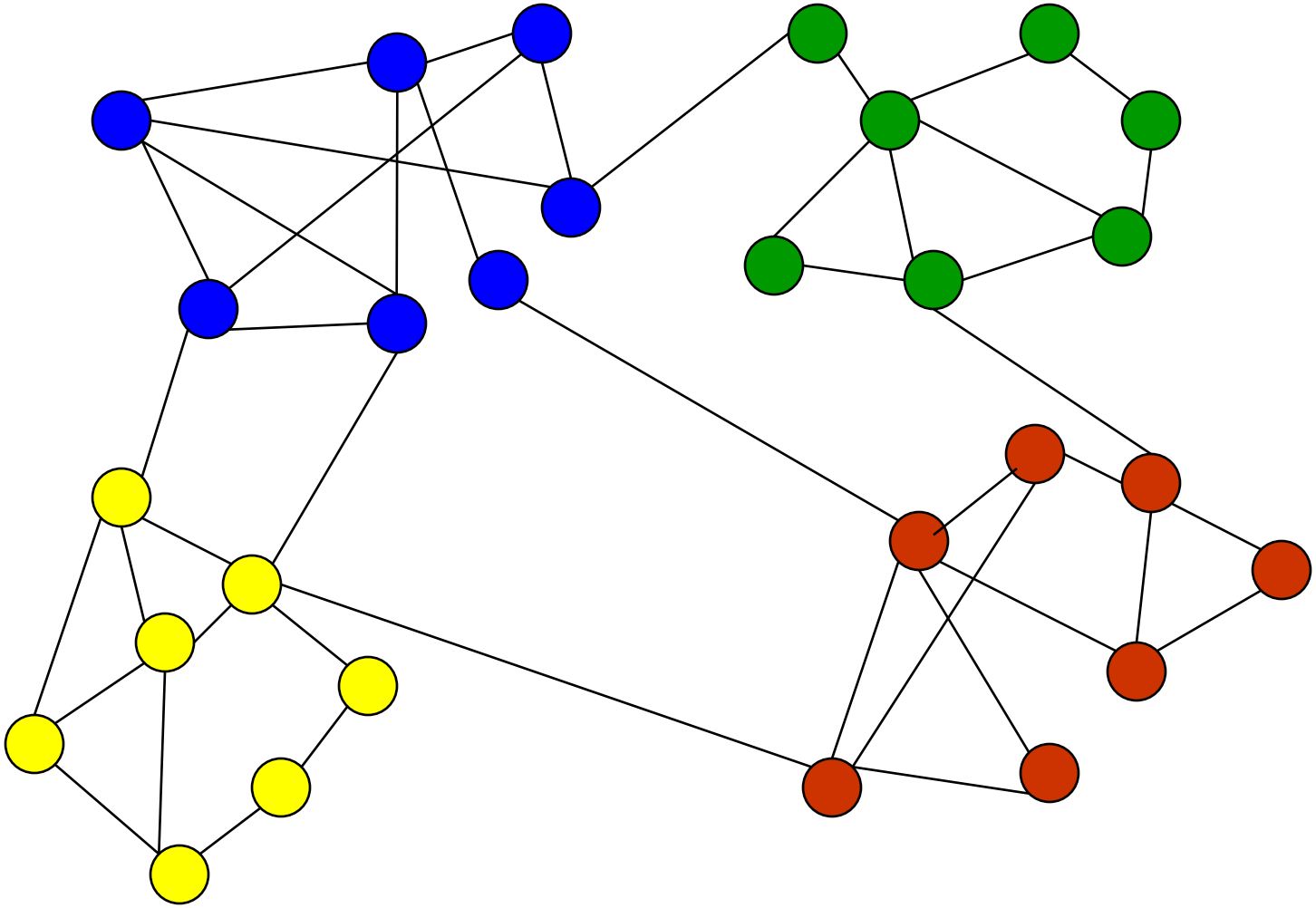
Learning/Updating Analysis (Golub Jackson 08)



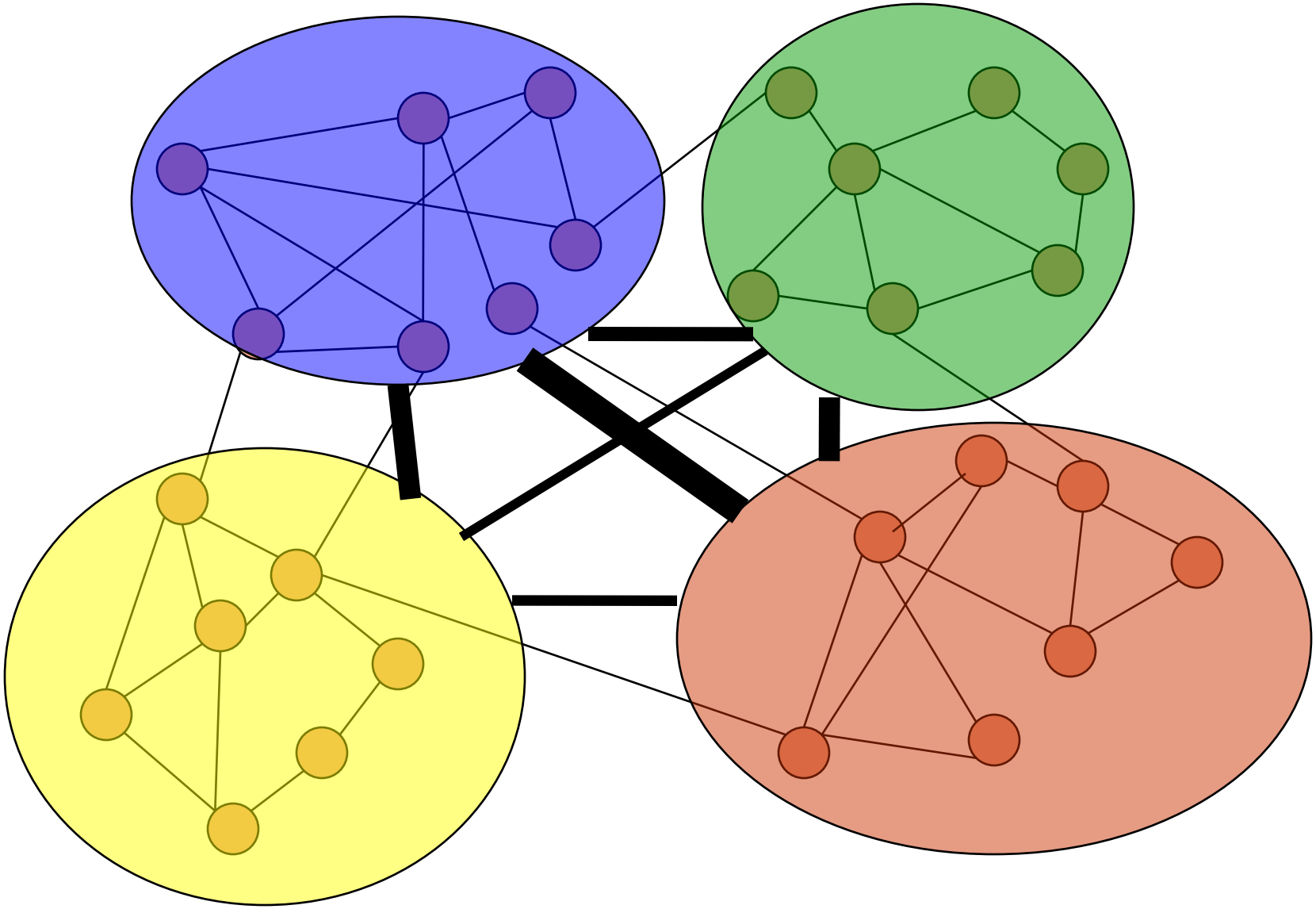
$CT(T(n),1/n) \approx \log(n) / \log(1/\text{Homophily})$

where Homophily = $(p_s - p_d) / (mp)$
is the extra probability of linking
to own type relative to
overall link probability

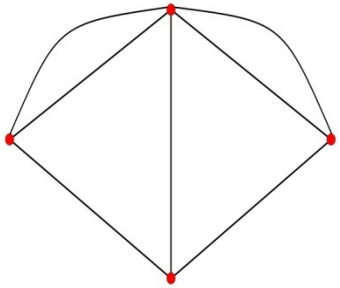
Representative Agents:



Representative Agents:



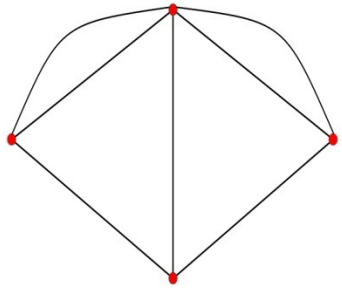
So



$$\text{AvgDist} \approx \log(n) / \log(d)$$

$$CT \approx \log(n) / \log(1/H)$$

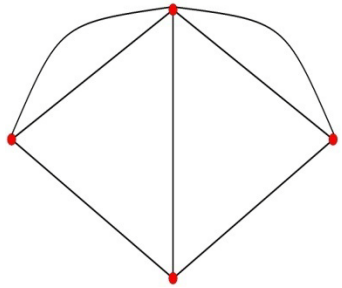
Speed:



Independent Variable

Process	Density	Homophily
Shortest Path	↑	0
Linear Updating	0	↓

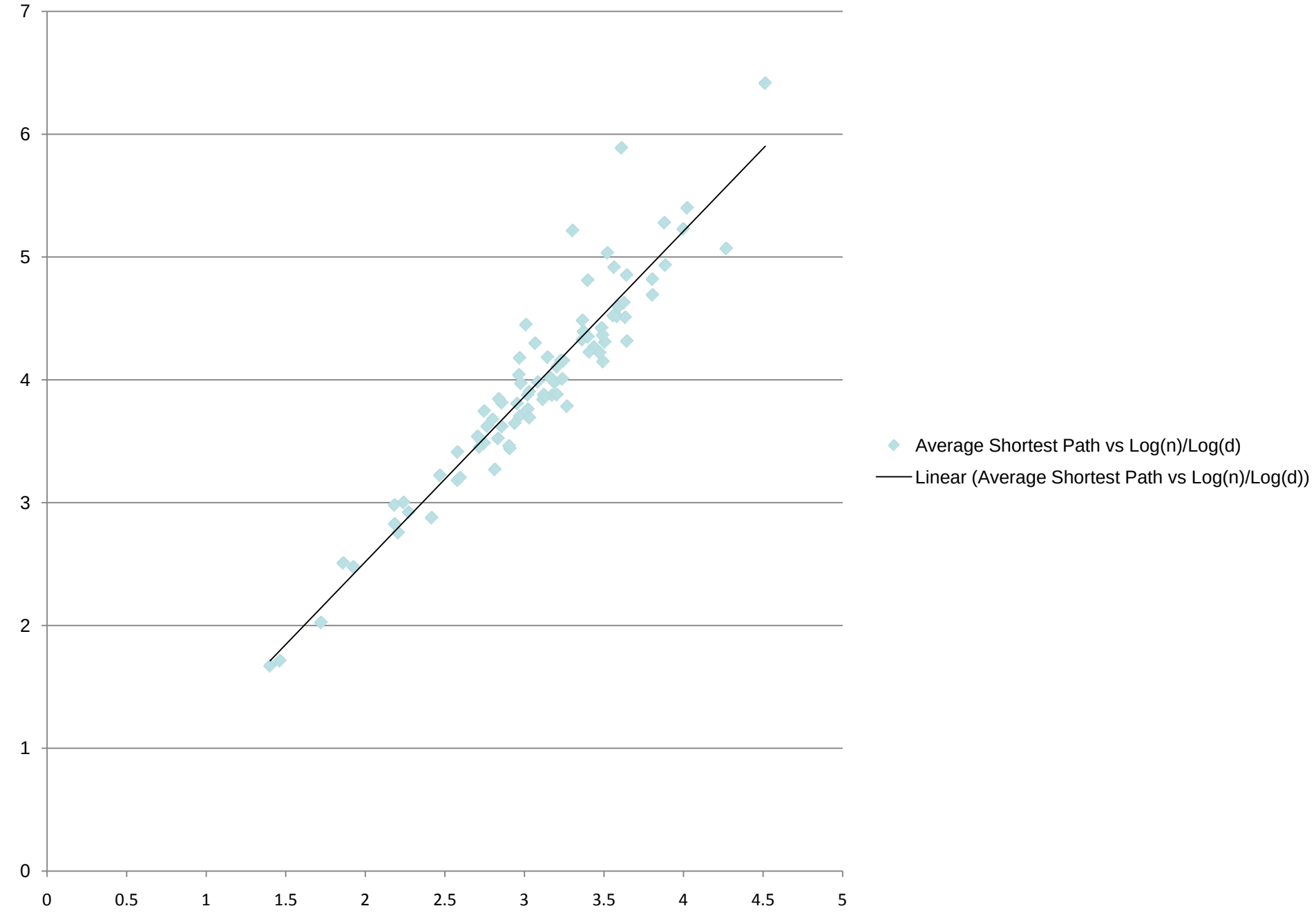
Summary

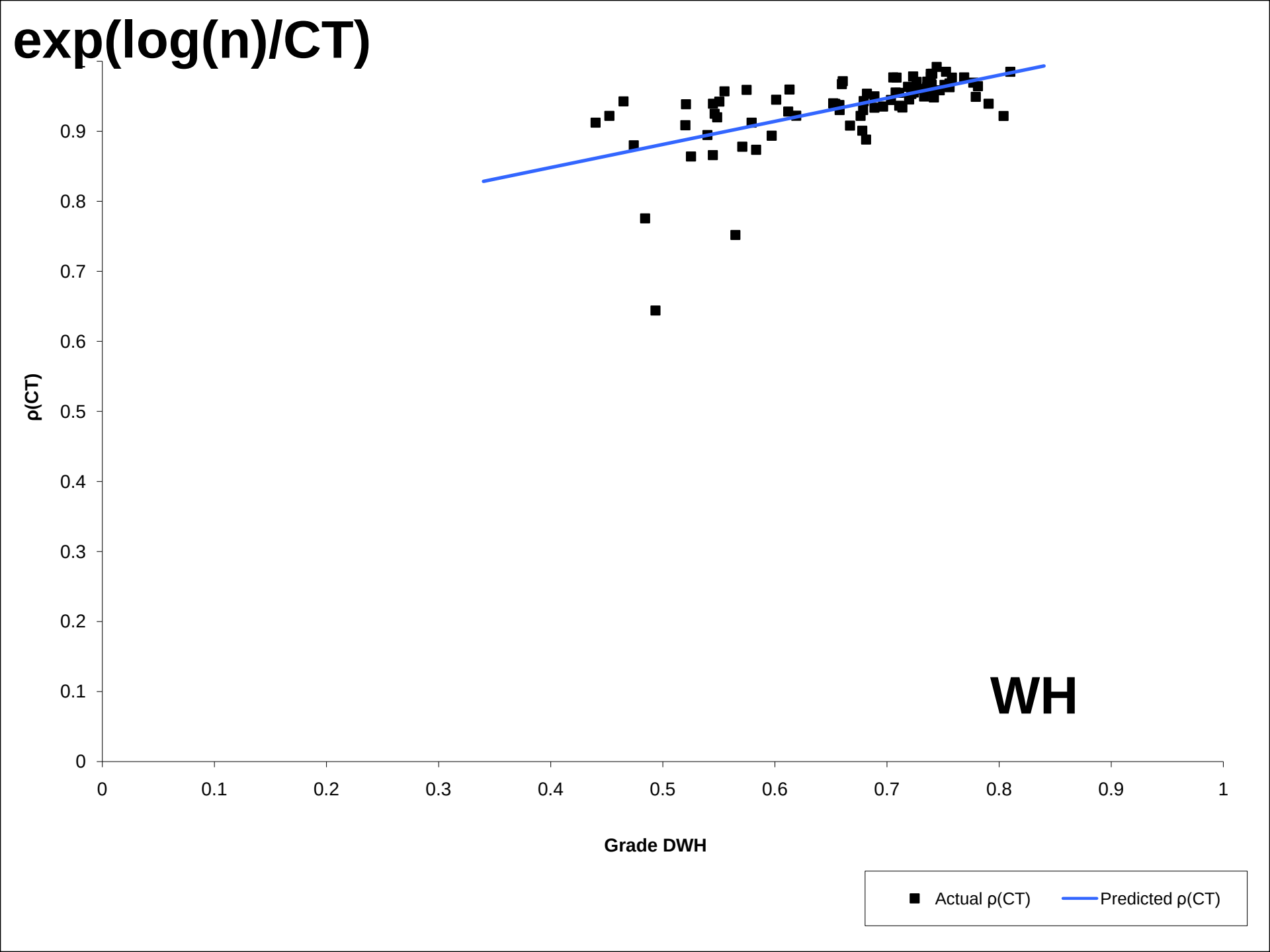


- Homophily
  - Slows learning based on updating from repeated averaging of neighbors' opinions
  - does not change diffusion/average path

- Other behavioral models:
  - Adjusting weights to beliefs?
  - Heterogeneity in Priors?
  - Choice Behavior…
- Other network characteristics…

Average Shortest Path vs Log(n)/Log(d)




exp(log(n)/CT)
ρ(CT)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Grade DWH
WH
Actual ρ(CT)
Predicted ρ(CT)